

Hybrid Parameter Estimation for GARCH-X Models via Approximate Bayesian Computation and Deep Learning

Bastían Díaz

Ingeniería Estadística, DMCC, USACH

Francisco Plaza Vega

DMCC, USACH

Abstract

This study proposes a hybrid approach for parameter estimation in GARCH-X models under scenarios with intractable likelihoods, integrating Deep Learning techniques with Approximate Bayesian Computation (ABC). GARCH-X models extend the GARCH family by incorporating exogenous covariates, which are relevant in systems where environmental variability shapes the observed dynamics. However, traditional estimation based on maximum likelihood becomes limited when innovations follow non-Gaussian distributions or when the likelihood is difficult to compute. In this context, ABC emerges as a viable alternative, but its performance critically depends on the construction of summary statistics.

To address this challenge, deep neural networks (CNN, CNN+LSTM, and TCN+LSTM with attention mechanisms) are trained using simulated trajectories of the GARCH-X process to learn informative summary statistics capable of predicting the model parameters. These statistics are then integrated into different ABC schemes, including rejection ABC and ABC-MCMC. The quality of parameter recovery is assessed using the coefficient of determination across one thousand replications of the estimation procedure.

Simulation results show that deep architectures significantly outperform manually designed summary statistics. In particular, the TCN+LSTM model with attention achieves the highest levels of accuracy. Among the ABC algorithms, ABC-MCMC displays greater stability when dealing with multimodal posterior distributions. Applied to monthly anchovy and sardine landing time series from northern Chile (1963–2023), the hybrid approach confirms the presence of volatility clusters and a high sensitivity to environmental variables such as ENSO indices and thermal gradients, revealing ecological mechanisms associated with population alternation.

The study demonstrates that combining Deep Learning and ABC provides a robust and generalizable tool for inference in GARCH-X models, offering new quantitative resources for analyzing fishery dynamics under climate variability.

Reference (optional)

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