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An efficient class of estimators using two auxiliary attributes

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Abstract

This paper proposes a class of estimators based on information of two auxiliary attributes. The expressions of mean square errors of the proposed class of estimators are derived in a general form. It is shown that the proposed class of estimators is always more efficient than regression estimator based on two attributes, estimators recently proposed by Verma et al. (2013) and Malik and Singh (2013). In addition, we support this theoretical result by an empirical study using original data to show the superiority of the constructed estimators over others.

Keywords: Attribute · point bi-serial · pi-correlation · mean square error · simple random sampling.

Mathematics Subject Classification: Primary 62D05.

1. Introduction

In the theory of sample surveys, it is usual to make use of the auxiliary information at the estimation stage in order to improve the precision or accuracy of an estimator of unknown population parameter of interest. Sometimes there exist situations when information is available in the form of attributes, which is highly correlated with study variable y. Several authors including Naik and Gupta (1996), Jhajj et al. (2006), Shabbir and Gupta (2007), Singh et al. (2008), Abd-Elafattah et al. (2010), Koyuncu (2012), Singh and Solanki (2012) Sharma et al. (2013a,b) and Malik and Singh (2014) proposed a set of estimators, taking the advantage of point bi-serial correlation between auxiliary attribute and study variable, using information on a single auxiliary attribute. In most of the cases, we see that instead of one auxiliary attribute, information of two qualitative variables are available. For instance, to estimate the hourly wages we can use the information on marital status and region of resident (see, Gujrati and Sangeeta, 2007). In such situations both auxiliary attributes have significant point bi-serial correlation with study variable and there is significant pi-correlation between the two auxiliary attributes. Verma et al. (2013) Malik and Singh

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(2013) and Sharma and Singh (2014), proposed some estimators using information on two auxiliary attribute in simple random sampling.

Consider a sample of size n drawn by simple random sampling without replacement (SRSWOR) from a population of size N. Let y_i and ϕ_{ij} (i=1,2) denote the observations on variable y and ϕ_i $(i = 1, 2)$ for the j^{th} unit $(j = 1, 2, \ldots, N)$. We note that

$$\phi_{ij} = \begin{cases} 1, & \text{if } i^{th} \text{ unit posses attributes.} \\ 0, & \text{otherwise.} \end{cases}$$

Let

$$A_i = \sum_{j=1}^{N} \phi_{ij} \qquad \text{and} \qquad a_i = \sum_{j=1}^{n} \phi_{ij},$$

for $i = 1, 2$ denotes the total number of units in the population and sample possessing attribute ϕ_i respectively. Similarly, $P_i = A_i/N$ and $p_i = a_i/n$ denotes the proportion of units in the population and sample, respectively, possessing attribute ϕ_i .

Let us define,

$$e_0 = \frac{(\bar{y} - \bar{Y})}{\bar{Y}}, \qquad e_1 = \frac{(p_1 - P_1)}{P_1} \qquad \text{and} \qquad e_2 = \frac{(p_2 - P_2)}{P_2}.$$

Such that $E(e_i) = 0$, $(i = 0, 1, 2)$

$$E(e_0^2) = f_1 C_y^2, \qquad E(e_1^2) = f_1 C_{p_1}^2, \qquad E(e_2^2) = f_1 C_{p_2}^2,$$
$$E(e_0 e_1) = f_1 K_{yp_1} C_{p_1}^2, \qquad E(e_0 e_2) = f_1 K_{yp_2} C_{p_2}^2, \qquad E(e_1 e_2) = f_1 K_{\phi} C_{p_2}^2,$$
$$K_{yp_1} = \rho_{yp_1} \frac{C_y}{C_{p_1}}, \qquad K_{yp_2} = \rho_{yp_2} \frac{C_y}{C_{p_2}}, \qquad K_{\phi} = \rho_{\phi} \frac{C_{p_1}}{C_{p_2}}.$$

where,

$$f_1 = \left(\frac{1}{n} - \frac{1}{N}\right), \qquad C_{p_j}^2 = \frac{S_{p_j}^2}{P_j^2}, \qquad (j = 1, 2).$$

2. Estimators in Literature

In order to have an estimate of the study variable y, using the information of population proportion P, Naik and Gupta (1996) and Singh et al. (2007) proposed the following estimator respectively

$$t_a = \bar{y}\frac{P_1}{p_1} \quad (1)$$

$$t_b = \bar{y}\frac{p_2}{P_2} \quad (2)$$

$$t_c = \bar{y}\ \exp\left(\frac{P_1 - p_1}{P_1 + p_1}\right) \quad (3)$$

$$t_d = \bar{y}\ \exp\left(\frac{p_2 - P_2}{p_2 + P_2}\right) \quad (4)$$

The MSE expressions of the estimators t_a, t_b, t_c and t_d are respectively given as

$$MSE(t_a) = f_1[\bar{Y}C_y^2 + C_{p_1}^2(1 - 2K_{yp_1})] \quad (5)$$

$$MSE(t_b) = f_1[\bar{Y}C_y^2 + C_{p_2}^2(1 + 2K_{yp_2})] \quad (6)$$

$$MSE(t_c) = f_1\left[\bar{Y}C_y^2 + C_{p_1}^2\left(\frac{1}{4} - K_{yp_1}\right)\right] \quad (7)$$

$$MSE(t_d) = f_1\left[\bar{Y}C_y^2 + C_{p_2}^2\left(\frac{1}{4} + K_{yp_2}\right)\right] \quad (8)$$

The regression estimator for estimating the unknown population mean of y, when information on an auxiliary attribute say p, correlated with study variable y, is available

$$t_{r_1} = \bar{y} + b(P - p), \quad (9)$$

where b is an estimate of the change in y when p is increased by unity.

The MSE expression of regression estimator using an auxiliary attribute is

$$MSE(t_{r_1}) = f_1\bar{Y}^2C_y^2(1 - \rho^2), \quad (10)$$

where C_y and C_p are the coefficients of variation of the variates y and p respectively.

When there are two auxiliary attributes P_1 and P_2, the regression estimator of population mean is

$$t_{r_2} = \bar{y} + b_1(P_1 - p_1) + b_2(P_2 - p_2), \quad (11)$$

where $b_1 = \frac{s_{yp_1}}{s_{p_1}^2}$ and $b_2 = \frac{s_{yp_2}}{s_{p_2}^2}$. $s_{p_1}^2$ and $s_{p_2}^2$ are the sample variances of p_1 and p_2 respectively , s_{yp_1} and s_{yp_2} are the sample covariance between y and p_1 and p_2 respectively.

The MSE expression of the t_{r_2} is

$$MSE(t_{r_2}) = f_1\bar{Y}^2C_y^2(1 - \rho_{yp_1}^2 - \rho_{yp_2}^2 + 2\rho_{yp_1}\rho_{yp_2}\rho_\phi). \quad (12)$$

Verma et al. (2013) proposed following three estimators using two auxiliary attributes

$$t_{v_1} = \bar{y}\left[K_{51}\frac{P_1}{p_1} + K_{52}\frac{P_2}{p_2}\right], \quad (13)$$

where K_{51} and K_{52} are constants such that, $K_{51} + K_{52} = 1$.

The minimum MSE of estimator t_{v_1} is given as

$$MSE(t_{v_1}) = f_1\bar{Y}^2(C_y^2 + K_{51}^2C_{p_1}^2 + K_{52}^2C_{p_2}^2 - 2K_{51}K_{yp_1}C_{p_1}^2 - 2K_{52}K_{yp_2}C_{p_2}^2 + 2K_{51}K_{52}K_\phi C_{p_2}^2),$$

where

$$K_{51} = \frac{K_{yp_1}C_{p_1}^2 - K_\phi C_{p_2}^2}{C_{p_1}^2 - K_\phi C_{p_2}^2} \quad \text{and} \quad K_{52} = 1 - K_{51}.$$

And

$$t_{v_2} = [K_{61}\bar{y} + K_{62}(P_1 - p_1)]\exp\left[\frac{P_2 - p_2}{P_2 + p_2}\right],$$

where K_{61} and K_{62} are constants.

The minimum MSE of estimator t_{v_2} is given as

$$MSE(t_{v_2}) = K_{61}^2\bar{Y}^2G_1 + K_{62}^2P_1^2G_2 - 2K_{61}K_{62}P_1\bar{Y}G_3 + \bar{Y}^2(1 - 2K_{61}), \tag{14}$$

where

$$G_1 = 1 + f_1\left(C_y^2 + C_{p_2}^2\left(\frac{1}{4} - K_{yp_2}\right)\right),$$

$$G_2 = f_1C_{p_1}^2 \quad \text{and} \quad G_3 = f_1\left(K_{yp_1}C_{p_1}^2 - \frac{1}{2}K_\phi C_{p_2}^2\right).$$

And

$$t_{v_3} = [\bar{y} + K_{71}(P_1 - p_1) + K_{72}(P_2 - p_2)],$$

where K_{71} and K_{72} are constants.

The minimum MSE of estimator t_{v_3} is given as

$$MSE(t_{v_3}) = f_1\left[\left(\bar{Y}^2C_y^2 + K_{71}^2P_1^2C_{p_1}^2 + K_{72}^2P_2^2C_{p_2}^2 - 2K_{71}P_1\bar{Y}K_{pb_1}C_{p_1}^2 - 2K_{72}P_2\bar{Y}K_{pb_2}C_{p_2}^2\right) + 2K_{71}K_{72}P_1P_2\bar{Y}K_\phi C_{p_2}^2\right], \tag{15}$$

where the optimum values of K_{71} and K_{72} are

$$K_{71} = \frac{\bar{Y}}{P_1}\left(\frac{K_{pb_1}C_{p_1}^2 - K_{pb_2}K_\phi C_{p_2}^2}{C_{p_1}^2 - K_\phi^2C_{p_2}^2}\right) = K_{71}^*$$

$$K_{72} = \frac{\bar{Y}}{P_2}\left(\frac{K_{pb_2}C_{p_1}^2 - K_{pb_1}K_\phi C_{p_1}^2}{C_{p_1}^2 - K_\phi^2C_{p_2}^2}\right) = K_{71}^*$$

Malik and Singh (2013) proposed a multivariate ratio estimator using information on two Auxiliary attributes

$$t_{M_1} = \bar{y}\left[\frac{m_1P_1 + m_2P_2}{m_1p_1 + m_2p_2}\right]^{\alpha} \quad (16)$$

where m_1 and m_2 are weights such that, $m_1 + m_2 = 1$.

The minimum MSE of estimator t_{M_1} is given as

$$\begin{aligned} MSE(t_{M_1}) = f_1(&C_y^2 + m_1^2\alpha^2\theta^2 S_{p_1}^2 + m_2^2\alpha^2\theta^2 S_{p_2}^2 - 2m_1\theta\alpha S_{yp_1} - 2m_2\theta\alpha S_{yp_2} \\ &+ 2m_1m_2\alpha^2\theta^2 S_{p_1p_2}), \end{aligned} \quad (17)$$

where

$$\theta = \frac{\bar{Y}}{m_1P_1 + m_2P_2}, \qquad m_1 = \frac{\alpha\theta S_{p_1p_2} - \alpha\theta S_{p_2}^2 - S_{yp_1} + S_{yp_2}}{\alpha\theta(S_{p_1}^2 + S_{p_2}^2 - 2S_{p_1P_2})} \qquad \text{and} \qquad m_2 = 1 - m_1.$$

Malik and Singh (2013) proposed an exponential type estimator as

$$t_{M_2} = \bar{y}\ \exp\left(\frac{P_1 - p_1}{P_1 + p_1}\right)^{\beta_1} \exp\left(\frac{P_2 - p_2}{P_2 + p_2}\right)^{\beta_2} \quad (18)$$

The minimum MSE of t_{M_2} is:

$$MSE(t_{M_2}) = \bar{Y}^2 f_1\left[C_y^2 + C_{p_1}^2\left(\frac{\beta_1^2}{4} - \beta_1 K_{pb_1}\right) + C_{p_2}^2\left(\frac{\beta_2^2}{4} + \frac{\beta_1\beta_2}{2}K_\phi - \beta_2 K_{pb_2}\right)\right] \quad (19)$$

where β_1 and β_2 are real constant.

Malik and Singh (2013) proposed another exponential type estimator as

$$t_{M_3} = \bar{y}\ \exp\left(\frac{P_1 - p_1}{P_1 + p_1}\right)^{\beta_1} \exp\left(\frac{P_2 - p_2}{P_2 + p_2}\right)^{\beta_2} + b_1(P_1 - p_1) + b_2(P_2 - p_2) \quad (20)$$

The minimum MSE of estimator t_{M_3} is given as

$$\begin{aligned} MSE(t_{M_3}) = \Bigg[&\bar{Y}^2\left\{C_y^2 + \frac{\beta_1^2 C_{p_1}^2}{4} + \frac{\beta_2^2 C_{p_2}^2}{4} - \beta_1 K_{yp_1}C_{p_1}^2 - \beta_2 K_{yp_2}C_{p_2}^2\right\} + B_1^2P_1^2C_{p_1}^2 \\ &+ B_2^2P_2^2C_{p_2}^2 + 2B_1B_2P_1P_2K_\phi C_{p_2}^2 - 2\bar{Y}\Bigg\{B_1P_1K_{yp_1}C_{p_1}^2 + B_2P_2K_{yp_2}C_{p_2}^2 \\ &- \frac{\beta_1B_1P_1C_{p_1}^2}{2} - \frac{\beta_2B_2P_2C_{p_2}^2}{2} - \frac{\beta_1B_2P_2K_\phi C_{p_2}^2}{2} - \frac{\beta_2B_1P_1K_\phi C_{p_2}^2}{2}\Bigg\}\Bigg], \end{aligned} \quad (21)$$

where

$$B_1 = \frac{S_{yp_1}}{S_{p_1}^2}, \qquad B_2 = \frac{S_{yp_2}}{S_{p_2}^2},$$

$$\beta_1 = \frac{A_1 - A_2K_\phi}{\bar{Y}(C_{p_1}^2 - K_\phi^2 C_{p_2}^2)} \qquad \text{and} \qquad \beta_2 = \frac{A_1 - \bar{Y}C_{p_1}^2\beta_1}{\bar{Y}K_\phi C_{p_2}^2}.$$

With

$$A_1 = 2\bar{Y}K_{yp_1}C_{p_1^2} - 2P_1B_1C_{p_1}^2 - 2P_2B_2K_\phi C_{p_2}^2 \qquad \text{and}$$
$$A_2 = 2\bar{Y}K_{yp_2}C_{p_2^2} - 2P_2B_2C_{p_2}^2 - 2P_1B_1K_\phi C_{p_2}^2.$$

3. The Proposed class of estimator

We propose another improved family of estimators for estimating $\bar{y}$ when information of two auxiliary attributes available, as

$$t_N = \bar{y}\left[w_1\left(\frac{p_1}{P_1}\right)^{\delta}\exp\left\{\frac{\eta_1(P_1-p_1)}{\eta_1(P_1+p_1)+2\lambda_1}\right\} + w_2\left(\frac{p_2}{P_2}\right)^{\beta}\exp\left\{\frac{\eta_2(P_2-p_2)}{\eta_2(P_2+p_2)+2\lambda_2}\right\}\right] \quad (22)$$

where δ and β are constants that can takes values (0,1,-1) for designing different estimators; η_1, λ_1, η_2 and λ_2 are either real numbers or the function of the known parameters. w_1 and w_2 are suitable chosen constants to be determined such that mean square error (MSE) of the class of estimator t_N is minimum.

It is to be mentioned that

(i) For (w_1, w_2) =(1,0), the class of estimator t_N reduces to the class of estimator as

$$t_{NK} = \bar{y}\left\{\left(\frac{p_1}{P_1}\right)^{\delta}\exp\left(\frac{\eta_1(P_1-p_1)}{\eta_1(P_1+p_1)+2\lambda_1}\right)\right\} \quad (23)$$

(ii) For $(w_1, w_2) = (0, w_2)$, the class of estimator t_N reduces to the class of estimator as

$$t_{NR} = \bar{y}\left\{w_2\left(\frac{p_2}{P_2}\right)^{\beta}\exp\left(\frac{\eta_2(P_2-p_2)}{\eta_2(P_2+p_2)+2\lambda_2}\right)\right\} \quad (24)$$

A set of new estimators generated from (25) using suitable values of δ, β, η_1, η_2, λ_1 and λ_2 are listed in Table 2.

Table 1. Set of estimators generated from the estimator t_N

Subset of proposed estimator	δ	η_1	λ_1	β	η_2	λ_2
$t_{N_1} = \bar{y}\left[w_1\left(\frac{p_1}{P_1}\right)\exp\left\{\frac{(P_1-p_1)}{(P_1+p_1)+2}\right\} + w_2\left(\frac{p_2}{P_2}\right)\exp\left\{\frac{(P_2-p_2)}{(P_2+p_2)+2}\right\}\right]$	1	1	1	1	1	1
$t_{N_2} = \bar{y}\left[w_1\exp\left\{\frac{(P_1-p_1)}{(P_1+p_1)+2}\right\} + w_2\exp\left\{\frac{(P_2-p_2)}{(P_2+p_2)+2}\right\}\right]$	0	1	1	0	1	1
$t_{N_3} = \bar{y}\left[w_1\left(\frac{p_1}{P_1}\right)\exp\left\{\frac{(P_1-p_1)}{(P_1+p_1)+2P_1}\right\} + w_2\left(\frac{p_2}{P_2}\right)\exp\left\{\frac{(P_2-p_2)}{(P_2+p_2)+2P_2}\right\}\right]$	1	1	P_1	1	1	P_2
$t_{N_4} = \bar{y}\left[w_1\left(\frac{p_1}{P_1}\right)\exp\left\{\frac{P_1(P_1-p_1)}{P_1(P_1+p_1)+2}\right\} + w_2\left(\frac{p_2}{P_2}\right)\exp\left\{\frac{P_2(P_2-p_2)}{P_2(P_2+p_2)+2}\right\}\right]$	1	P_1	1	1	P_2	1
$t_{N_5} = \bar{y}\left[w_1\left(\frac{p_1}{P_1}\right)\exp\left\{\frac{C_{p_1}(P_1-p_1)}{C_{p_1}(P_1+p_1)+2P_1}\right\} + w_2\left(\frac{p_2}{P_2}\right)\exp\left\{\frac{C_{p_2}(P_2-p_2)}{C_{p_2}(P_2+p_2)+2P_2}\right\}\right]$	1	C_{p_1}	P_1	1	C_{p_2}	P_2
$t_{N_6} = \bar{y}\left[w_1\left(\frac{P_1}{p_1}\right)\exp\left\{\frac{P_1(P_1-p_1)}{P_1(P_1+p_1)+2}\right\} + w_2\left(\frac{p_2}{P_2}\right)\exp\left\{\frac{P_2(P_2-p_2)}{P_2(P_2+p_2)+2}\right\}\right]$	-1	P_1	1	1	P_2	1
$t_{N_7} = \bar{y}\left[w_1\left(\frac{P_1}{p_1}\right)\exp\left\{\frac{P_1(P_1-p_1)}{P_1(P_1+p_1)+2}\right\} + w_2\left(\frac{P_2}{p_2}\right)\exp\left\{\frac{P_2(P_2-p_2)}{P_2(P_2+p_2)+2}\right\}\right]$	-1	P_1	1	-1	P_2	1
$t_{N_8} = \bar{y}\left[w_1\exp\left\{\frac{P_1(P_1-p_1)}{P_1(P_1+p_1)+2}\right\} + w_2\left(\frac{P_2}{p_2}\right)\exp\left\{\frac{P_2(P_2-p_2)}{P_2(P_2+p_2)+2}\right\}\right]$	0	P_1	1	-1	P_2	1
$t_{N_9} = \bar{y}\left[w_1\exp\left\{\frac{P_1(P_1-p_1)}{P_1(P_1+p_1)+2}\right\} + w_2\left(\frac{p_2}{P_2}\right)\exp\left\{\frac{P_2(P_2-p_2)}{P_2(P_2+p_2)+2}\right\}\right]$	0	P_1	1	1	P_2	1
$t_{N_{10}} = \bar{y}\left[w_1\exp\left\{\frac{P_1(P_1-p_1)}{P_1(P_1+p_1)+2}\right\} + w_2\exp\left\{\frac{P_2(P_2-p_2)}{P_2(P_2+p_2)+2}\right\}\right]$	0	P_1	1	0	P_2	1

Expressing the class of estimators t_N at equation (25) in terms of e's, we have

$$t_N = \bar{Y}(1+e_0)\left[w_1(1+e_1)^{\delta}\left\{1+\gamma_1 e_1+\frac{3}{2}\gamma_1^2 e_1^2\right\}\right.$$
$$\left. + w_2(1+e_2)^{\beta}\left\{1+\gamma_2 e_2+\frac{3}{2}\gamma_2^2 e_2^2\right\}\right] \quad (25)$$

where

$$\gamma_1 = \frac{\eta_1 P_1}{2(\eta_1 P_1+\lambda_1)} \qquad \text{and} \qquad \gamma_2 = \frac{\eta_2 P_2}{2(\eta_2 P_2+\lambda_2)}.$$

Simplifying equation (28) and retaining terms to the first order of approximation, we have

$$(t_N-\bar{Y}) = \bar{Y}[w_1(1+e_0-A(e_1+e_0e_1)+De_1^2)$$
$$+w_2(1+e_0-C(e_2+e_0e_2)+De_2^2)-1,] \quad (26)$$

where

$$A=\delta-\gamma_1, \qquad B=\frac{3}{2}\gamma_1^2-\delta\gamma_1+\frac{\delta(\delta-1)}{2},$$
$$C=\beta-\gamma_2 \qquad \text{and} \qquad D=\frac{3}{2}\gamma_2^2-\beta\gamma_2+\frac{\beta(\beta-1)}{2}.$$

Squaring both sides of equation (29) and taking expectations of both sides, we get the MSE of the estimator t_N to the first order of approximation, as

$$MSE(t_N) = \bar{Y}^2[1+w_1^2A_1+w_2^2A_2-2w_1A_3-2w_2A_4+2w_1w_2A_5] \quad (27)$$

where

$$A_1 = \left\{1+f_1(C_y^2+C_{p_1}^2(A^2+2B-4AK_{yp_1}))\right\}$$
$$A_2 = \left\{1+f_1(C_y^2+C_{p_2}^2(C^2+2D-4CK_{yp_2}))\right\}$$
$$A_3 = \left\{1-f_1C_{p_1}^2(AK_{yp_1}-B)\right\}$$
$$A_4 = \left\{1-f_1C_{p_2}^2(CK_{yp_2}-D)\right\}$$
$$A_5 = \left\{1+f_1(C_y^2+C_{p_1}^2(B-2AK_{yp_1})+C_{p_2}^2(D-2CK_{yp_2}))\right\}$$

The MSE of the class of estimator t_N at equation (30) is minimised for the optimum values of w_1 and w_2 given as

$$w_1^* = \frac{(A_2A_3-A_4A_5)}{A_1A_2-A_5^2} \qquad \text{and} \qquad w_2^* = \frac{(A_1A_4-A_3A_5)}{A_1A_2-A_5^2}.$$

The minimum MSE of estimator t_N is

$$MSE(t_N) = \bar{Y}^2[1+w_1^{*2}A_1+w_2^{*2}A_2-2w_1^*A_3-2w_2^*A_4+2w_1^*w_2^*A_5] \quad (28)$$

4. Empirical Study

Population I

The data used for empirical study has been taken from government of Pakistan (2004) for the population consists rice cultivation areas in 73 districts of Pakistan. The variables are defined as:

Y: Rice production (in 000' tonnes during 2003).
P_1: Production of farms where rice production is more than 20 tonnes during the year 2002.
P_2: Production of farms where rice cultivation area is more than 20 hectares during the year 2003.

Using raw data we have calculated the following values.

$$N = 73, \quad n = 15, \quad \bar{Y} = 61.3, \quad P_1 = 0.4247, \quad P_2 = 0.3425,$$
$$S_y^2 = 12371.4, \quad S_{\phi_1}^2 = 0.2254, \quad S_{\phi_2}^2 = 0.2283,$$
$$\rho_{yp_1} = 0.621, \quad \rho_{yp_2} = 0.673, \quad \rho_{\phi} = 0.889.$$

Population II

The data used for empirical study has been taken from Singh and Chaudhary (1986, p. 177) for the population consists of 34 wheat farms in 34 villages in certain region of India. The variables are defined as:

Y: Area under wheat crop (in acres) during 1974.
P_1: Proportion of farms under wheat crop which have more than 500 acres land during 1971.
P_2: Proportion of farms under wheat crop which have more than 100 acres land during 1973.

Using raw data we have calculated the following values.

$$N = 34, \quad n = 15, \quad \bar{Y} = 199.4, \quad P_1 = 0.6765, \quad P_2 = 0.7353,$$
$$S_y^2 = 22564.6, \quad S_{\phi_1}^2 = 0.225490, \quad S_{\phi_2}^2 = 0.200535,$$
$$\rho_{yp_1} = 0.599, \quad \rho_{yp_2} = 0.559, \quad \rho_{\phi} = 0.725.$$

The following Table shows comparison between some existing estimators and proposed estimators with respect to usual estimator.

Table 2 exhibits that the estimators based on auxiliary attributes are more efficient than the one ($\bar{y}$) which does not utilize the auxiliary information. Most of the members of the proposed family of estimators t_N are more efficient than the estimators considered here. Scrupulously, estimator t_{N_6} of proposed class of estimator t_N is best among them.

Table 2. Variances / MSEs / minimum MSEs and PRE's of different Estimators

	Population I		Population II	
Estimator	MSE	PRE	MSE	PRE
$\bar{y}$	655.34	100.00	840.64	100.00
t_a	402.59	162.76	632.11	132.99
t_b	1343.14	48.78	2579.79	32.59
t_c	497.97	131.58	518.45	162.13
t_d	1087.05	60.28	1357.89	61.91
t_{r_1}	402.58	142.16	539.02	155.96
t_{r_2}	598.19	109.32	684.48	122.81
t_{v_1}	395.04	165.87	617.37	136.17
t_{v_2}	331.45	197.70	512.71	163.96
t_{v_3}	572.23	114.92	474.89	177.02
t_{M_1}	358.67	182.69	535.29	157.04
t_{M_2}	383.52	170.62	579.14	145.19
t_{M_3}	357.61	183.56	511.45	164.36
t_{N_1}	305.52	214.20	546.82	153.73
t_{N_2}	647.10	101.20	1384.44	60.62
t_{N_3}	247.36	264.91	491.50	171.04
t_{N_4}	267.62	205.95	433.05	194.04
t_{N_5}	318.17	244.85	390.42	215.32
t_{N_6}	109.54	598.21	259.90	323.43
t_{N_7}	777.69	84.34	2226.06	37.76
t_{N_8}	459.44	142.65	1130.71	74.35
t_{N_9}	359.65	182.20	444.02	189.33
$t_{N_{10}}$	267.58	244.64	1152.37	72.99

5. Conclusion

This paper proposed a family of estimators for estimating unknown population mean using information on two auxiliary attributes. Moreover, it was found that the proposed family of estimators were more efficient than the estimators which utilize information on single attribute (t_a, t_b, t_c and t_d) , estimators of Verma et al. (2013) and Malik and Singh (2013).

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